OPTIMAL DESIGN OF TWO-STAGE SPEED REDUCER SHAFTS

Abstract: *In this paper the optimal design of the shafts of a two stage-speed reducer is presented. As main input data we considered the gearings (obtained in an optimal manner, and which represent the subject of another research of Intelligentics). For this interesting example of optimal design seven genes were taken into consideration and a set of* 43 *constraints were formulated. Note that all the seven genes represent full standardized features: two shaft ends, two radial seals, and three radial taper bearings. The constraints refer to shafts strength* (3 *constraints*)*, bending and torsion deformations* (14)*, shafts fatigue* (21)*, bearings life* (3)*, keys strength* (8)*, and mounting/dismounting conditions* (12)*. In solving the optimization problem we used an original two-phase evolutionary algorithm* (2*PhEA*) *inspired from the evolutionary concept of "punctuated equilibrium".* 2*PhEA is implemented in Kreator v.*3.4 *software*

Keywords*: evolutionary algorithms, reducer shafts, optimal design*

1. INTRODUCTION

With the development of modern industry, machine design is higher required, especially the optimization design for parameters of mechanical parts, such as: mass, stress and transmission precision. That why one of the important novelty of this article consists in the optimal approach of the design of two-stage speed reducer shafts. It is know that a reducer is an important mechanical part which is widely used in aerospace, automobile, lathe and so on. Many researchers have reported solutions to this problem. In [9], Li et al presented an optimal design problem of a two-stage speed reducer. The purpose of the paper is to obtain the multi-objective optimization design scheme of a gear reducer with a Fuzzy Genetic Algorithm (FGA). However they use a set of only 7 constraints too little related to the design realities. Li and Symmons in [10] performed the optimized design of helical gear reducer using the minimum centre distance as the objective function. In [13], [14] and [15] an optimal design problem of a one stage speed reducer is presented. In these papers the objective was to minimize the weight of the reducer. In [13], [14] and [15] the authors used a set of seven variables as follows: face width, module of teeth, number on pinion teeth, length of input shaft between the bearings, length of output shaft between bearings, diameter of shaft 1 and diameter of shaft 2. From the above listed seven variables four are referring to the reducer shafts. The objective function was subjected to a simply formulated (from a mechanical point of view) set of 11 constraints. Mezura in [13] utilized this problem only to test one of previous version of his Simple Multimembered Evolution Strategy (SMES) software. In this paper the approach chosen by us for the design problem makes it one of the most round works in comparison to all above mentioned works. We dealt separately with the gearing (the reducer gearing were the subject of another research of Intelligentics) and the shafts. For optimal design of the reducer shafts which represent the topic of this work, the main input data were considered the gearings (obtained in an optimal manner also by Intelligentics).

2. TWO PHASE ENHANCED EVOLUTIONARY ALGORITHM FORMAT

Optimization problems with a very large number of constraints can be very difficult to solve. In order to remove this shortcoming, our two-phase enhanced evolutionary algorithm inspired from the evolutionary concept of *punctuated equilibrium* is used in this work.

Punctuated equilibrium [12] is a theory about how new species evolve that was first advances by paleontologists Niles Eldridge and Stephan Jay Gould in 1972 [4]. Before punctuated equilibrium, most scientists assumed that evolutionary change occur slowly and continuously in almost all species, and that new species originate either by slow divergence of small, isolated groups or by slow evolutionary transformation of whole species. But studies of the fossil records have shown that the biological evolution is a strong non-equilibrium process with long periods of stasis interrupted by avalanches of large changes in biosphere. According to the proponents of punctuated equilibrium [4], [5], for the majority of time species are in evolutionary stasis, with little or no change occurring and hence little or no increase in adaptation to their environments. Occasionally, often due to some environmental catastrophe (or planetwide climatic change [6]), there will be punctuations, periods of rapid evolutionary change during which speciations occur. So, punctuationists claim that (i) except when speciation occurs, species are in stasis and do not become increasingly adapted to their environments, and (ii) gradual natural selection alone is insufficient for speciation, which requires a punctuation event. Therefore the biological evolution can be considered as a kind of self-organized criticality (SOC) dynamics and, therefore, SOC gives an insight into emergent complexity in nature. Bak [1] contended that the critical state was the *most efficient state that can actually be reached dynamically*, and in this state, a population in an apparent equilibrium evolves episodically in spurts. Local change may affect any other element in the system, and this delicate balance arises without any external, organizing force. In other words, in terms of evolutionary computation, evolution of a species consists of exaptations of jumping from one hilltop to another nearby in some fitness landscape. Naturally such jumps will be rare, separated by large time intervals where species are located at a fitness peak, and the resulting evolutionary pattern will show punctuations as indeed seen in the fossil record [2]. Probably punctuated equilibrium is the best known example of evolutionary metastability [3]. From the beginning, the theory of punctuated equilibrium has inspired many computational approaches. Hereinafter we presented some significant results. Bornholdt and Sneppen [2] have studied evolution of a single genetic network, ideally representing a single species, in the absence of any competition. The evolution is driven by a noisy environment and the evolutionary step consists of random mutations combined with selection of mutants preserving the phenotype with respect to a given environment. Thus, the only requirement in this minimalistic model is continuity in phenotype. This simplification allowed them to discuss how the requirement of evolving robust networks in itself may lead to an evolution which exhibits punctuated equilibrium. Jonnal and Chemero [7] describe experiments in artificial life in which a neural network is artificially evolved to control a virtual creature. With the evolutionary algorithm employed in the artificial evolution, it was possible to simulate punctuated equilibrium: in some trials, instead of keeping the overall rate of mutations μ constant for the entire trial, they introduced a probability p that μ increased by some factor m over the course of a trial, so that for an individual generation, there is probability p that the mutation rate is set to $m\mu$. In all but one case, the trials that included occasional punctuations had final fitness scores that were better than the scores of the trials that had no punctuations. Lewis et al [8] utilized the punctuated equilibrium concept in developing a new Evolutionary Programming algorithm that, in addition to the conventional mutation and selection operations, implements a further selection operator to encourage the development of a SOC system. The algorithm is evaluated on a range of test cases drawn from real-world problems, and compared against a selection of algorithms, including gradient descent, direct search methods and a genetic algorithm. The results were very encouraging. Martz et al [11] used genetic algorithms in order to design reliability experiments. Genetic algorithms were executed in batches of 100 generations in order to allow for punctuated equilibrium. The best 10 solutions after a given batch has been completed become the initial set of designs for the next batch of 100 generations. After several batches of 100 generations of solutions have been obtained in this way, we finally report the design having the highest utility as our desired near-optimal Bayesian experimental design. Intelligentics have a totally different point of view on implementing the concept of punctuated equilibrium in an evolutionary optimization algorithm. We think that the high level of stress in the population (which determines sudden and massive changes of the species) is comparable to the effect of constrains of an optimization problem. Therefore, the main idea behind our 2PhEA algorithm is its operation in two phases. In each phase, the individual's fitness is determined by another factor. In *Phase 1*, the individual's fitness depends only on the way in which an individual is more suitable (or not) in terms of constraints. In this phase, the population "fight for survival" and there is no interest for the best individual. For this reason, the number and level of mutations is high, respectively very high. We thought this phase as some kind of "feasible individual generator". The algorithm moves into the second phase when the number of feasible individuals of the population exceeds a preset threshold. *Phase 2* is a common evolutionary algorithm (sometimes a simple genetic algorithm). In the following we present, in short, how to determine an individual's fitness in both phases of the algorithm. The optimization problem consists of an objective function f accompanied by certain number of constraints. The search space is considered the space of the n – dimensional decision vectors: $\bar{x} = \left(x^{(1)}, x^{(2)}, \ldots, x^{(n)}\right)$

where:

n – number of genes (variables);

The constraints of the problem are:

n_u – inequality type constraints:

$$g_i(\bar{x}) \le 0, \quad i = \overline{1, n_u}$$

n_s – strict inequality type constraints:

$$g_i(\bar{x}) < 0, \quad i = \overline{n_u + 1, n_u + n_s}$$

n_e – equality type constraints:

$$g_i(\bar{x}) = 0, \quad i = \overline{n_u + n_s + 1, n_u + n_s + n_e}$$

In order to use this constraints in our algorithm we needed to aggregate them in the following form:

$$G_i(\bar{x}) = \begin{cases} \left.\begin{array}{l} 0, g_i(\bar{x}) \le 0 \\ g_i(\bar{x}), g_i(\bar{x}) > 0 \end{array}\right\} i = \overline{1, n_u} \\ \left.\begin{array}{l} 0, g_i(\bar{x}) < 0 \\ g_i(\bar{x}) + \varepsilon, g_i(\bar{x}) \ge 0 \end{array}\right\} i = \overline{n_u + 1, n_u + n_s} \\ \left.\begin{array}{l} 0, g_i(\bar{x}) = 0 \\ |g_i(\bar{x})|, g_i(\bar{x}) \ne 0 \end{array}\right\} i = \overline{n_u + n_s + 1, n_u + n_s + n_e} \end{cases}$$

where:

ε – very small positive quantity.

In each phase, for each individual a so-called *score* is computed. The *partial score* of an individual (from those N individuals of the population) $\bar{x}_j$, $j = \overline{1, N}$, regarding to the constraint i, $i = \overline{1, n_u + n_s + n_e}$ is calculated as follows:

$$PS_i(\bar{x}_j) = G_i(\bar{x}_j) \Big/ \sum_{k=1}^{N} G_i(\bar{x}_k).$$

Eventually, the (*individual*) *score* of each individual $\bar{x}_j$, $j = \overline{1, N}$ of the population is:

$$S(\bar{x}_j) = \sum_{i=1}^{n_u + n_s + n_e} PS_i(\bar{x}_j)$$

Table 1. Values of genes

Ind #	Constraints 1		Constraints 2		Constraints 3		Ind Score	Rank phase 1
	Value	Partial Score	Value	Partial Score	Value	Partial Score		
1	52.3	0.17	22	0.07	512	0.31	0.55	4
2	31.2	0.10	37	0.12	831	0.50	0.72	5
3	0	0.00	0	0.00	0	0.00	**0.00**	**1**
4	211.0	0.69	0	0.00	294	0.18	0.87	7
5	0	0.00	8	0.03	0	0.00	0.03	3
6	0	0.00	0	0.00	0	0.00	**0.00**	**2**
7	11.8	0.04	253	0.79	18	0.01	0.84	6
Σ	306.3		320		1655			

Obviously, any feasible individual has null score. During *Phase 1* the population is sorted by the *score*, and during *Phase 2*, the population is sorted by *score* and *objective value*. In both phases the fitness of an individual is set according to its rank. 2PhEA is implemented in our Kreator software. In table 1, an example of a population of 7 individuals is presented, and it is explained how the rank of an individual is established according to its capacity to meet the constraints of the optimization problem. Not that the rank of feasible individuals (#2 and # 6 in this example) are randomly set. In order to implement such two-phase algorithm it is necessary to design some appropriate adjustable evolutionary features, even more because the two phases of the evolution require different tuning. This should not be considered a shortcoming, but an opportunity for fine control of evolution. The original evolutionary features discussed here are the fitness function and the genetic operators. In the design of the fitness function we consider two aims: (i) the function should be as simple as possible, even it is adjustable, and (ii) the adjustment should be made by meaning of a single parameter. With these in mind we propose here a *linear adjustable fitness function* with a single tuning parameter, the *selection pressure*. As it was already mentioned, we assume that the fitness Φ(*j*) of an individual *j*, $j = \overline{1, N}$ is set according to its rank in the sorted population and represents its probability of selection. We consider the *selection pressure SP* as the ratio of the best individual's *selection* probability to the average *selection* probability of all individuals in the *selection* pool (the whole population here). Since:

$\sum_{j=1}^{N} \Phi(j) = 1$, it results that: $\Phi(1) = SP/N$

The proposed *linear adjustable fitness function* has the following form:

$$\Phi(k)=\begin{cases} a_1 \cdot j + b_1, \text{ if } j=\overline{1,\, p} \\ a_2 \cdot j + b_2, \text{ if } j=\overline{p+1,\, N} \end{cases}$$

where:

$$a_1 = \left[2N - SP\cdot(N+p-1)\right]/N/(N-1)/(p-1)$$
$$b_1 = (p\cdot SP - 2)/(N-1)/(p-1)$$
$$a_2 = (p\cdot SP - 2N)/N/(N-p)/(p-1)$$
$$b_2 = (2N - p\cdot SP)/N/(N-p)/(N-1)$$

The value of the threshold p should be in the range $[p_{\min}, p_{\max}]$, where:

$$p_{\min} = 2N/SP - N + 1$$
$$p_{\max} = 2N/SP$$

Note that if $p_{\min}$ or $p_{\max}$ exceeds the bounds of the range, it will be set to the appropriate bound. In order to reduce the number of parameters, that could make difficult the tuning of the selection operator, we set the value of the threshold at:

$$p = \mathrm{round}\left[(p_{\min} + p_{\max})/2\right]$$

In fig. 1 the graph of fitness function is presented in the case of a selection pressure of 1.5. A similar or smaller value is used in *Phase 1*, when it is mandatory to not to prioritize none/any of the feasible individuals (in order to preserve the diversity of population). In *Phase 2* the selection pressure has to have a larger value. The more an elitist evolution is desirable, the larger the value of the selection pressure should be set.

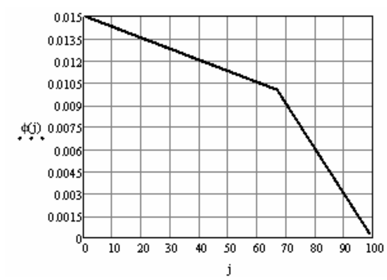

Fig. 1. Fitness function (N = 100, SP = 1.5, p = 67)

In fig. 2 the graph of a strong elitist fitness function is plotted for a selection pressure of 3.0.

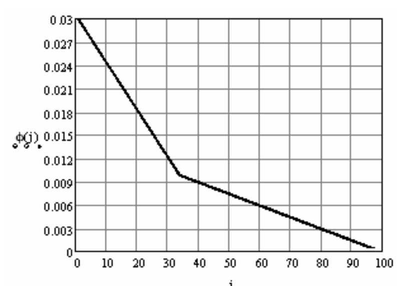

Fig. 2 Fitness function (N = 100, SP = 3.0, p = 34)

The above presented 2PhEA requests adjustable genetic operators for recombination and mutation, respectively. These two genetic operators are inspired by the *Monte Carlo random number generators* which generate normally distributed (statistically independent) numbers. In order to present the *normal recombination operator* let us consider two individuals $\overline{x}_1$ and $\overline{x}_2$ that will be mated, and assume that their k-th gene will suffer the recombination: $x_1^{(k)}, x_2^{(k)} \in \left[x_{lo}^{(k)}, x_{up}^{(k)}\right]$.

Let also assume that the parents satisfy the relationship $x_1^{(k)} \le x_2^{(k)}$. The obtained off-springs are:

$$y_1^{(k)} = x_1^{(k)} + \sigma_1 \cdot \sqrt{-2\ln u_2} \cdot \eta(u_1)$$
$$y_2^{(k)} = x_2^{(k)} + \sigma_2 \cdot \sqrt{-2\ln u_2} \cdot \eta(u_2)$$

where:

u_1, u_2 – random uniform distributed number on (0, 1), and

$$\eta(u) = \begin{cases} \sin(2\pi u_1), & \text{if } u \le 0.5 \\ \cos(2\pi u_1), & \text{if } u > 0.5 \end{cases}$$

$$\sigma_1 = q_c \cdot \min\left[x_1^{(k)} - x_{lo}^{(k)}, \left(x_2^{(k)} - x_1^{(k)}\right)/2\right]$$

$$\sigma_2 = q_c \cdot \min\left[x_{up}^{(k)} - x_2^{(k)}, \left(x_2^{(k)} - x_1^{(k)}\right)/2\right]$$

It is very easy to understand that $y_1^{(k)} \in N\left(x_1^{(k)}, \sigma_1\right)$ and $y_2^{(k)} \in N\left(x_2^{(k)}, \sigma_2\right)$ (where $N(\mu, \sigma)$ is a normal distribution with mean μ and variance σ^2). That means that each off-spring is part of a normal distribution with one of the parents as mean. The standard deviation is adjustable through the value of the parameter q_c. If the parameter q_c has a small value then the off-springs will be generated in the very neighborhood of the parents, and if q_c has higher value then the off-springs will be produced far away from their parents. For this reason in *Phase 1* we set the q_c parameter at higher values (closed to 1), and in *Phase 2* we used almost always $q_c = 1/3$.

The two off-springs $y_1^{(k)}, y_2^{(k)}$ should be in the range $\left[x_{lo}^{(k)}, x_{up}^{(k)}\right]$. If they exceed these bounds then their values will be trimmed. Regarding to the *natural mutation operator* we constructed it in the same manner as the recombination operator. If $y^{(k)} \in \left[x_{lo}^{(k)}, x_{up}^{(k)}\right]$ is one of the two off-springs obtained after recombination and which will suffer a mutation, then the mutant is given by the equation:

$$z^{(k)} = y^{(k)} + \sigma \cdot \sqrt{-2\ln u_2} \cdot \eta(u_1)$$

where:

$$\sigma = q_m \cdot \min\left(y^{(k)} - x_{lo}^{(k)}, x_{up}^{(k)} - y^{(k)}\right)$$

Obviously $z^{(k)} \in N\left(y^{(k)}, \sigma\right)$ and the mutant $z^{(k)}$ should be in the range $\left[x_{lo}^{(k)}, x_{up}^{(k)}\right]$. If the mutant $z^{(k)}$ exceeds these bounds then its value will be trimmed. The strategy of setting the parameter q_m is the same as those of the setting of q_c, and the used value were $q_m = 1/6 \ldots 1/3$.

In the following subsections, we outline the formulation of the optimal reducer shafts design problem in a systematic manner.

3. DESIGN PROBLEM FORMULATION

The aim of our optimal design is to minimize the mass of the three shaft sub-assemblies (including the mass of the shafts, of gearing and of six tapered rolling bearings) for the following input data:

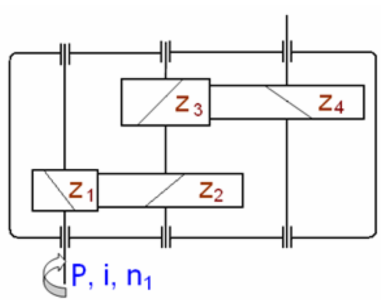

Fig. 3. The reducer sketch

Electrical engine horsepower:

$P = 2.9$ kW;

Overall transmission ratio:

$i = 7.6$;

Rotational speed of first shaft:

$n_1 = 925$ rpm

4. OPTIMAL DESIGN OF TWO-STAGE SPEED REDUCER SHAFTS

In order to perform the optimal design some preliminary phases have to be firstly completed:

- Identification of the variables (genes) that uniquely describe the problem (both the objective function and the constraints);
- Establishment of the objective function;
- Identification of the problem constraints.

Optimization problem genes

The authors consider that are seven genes that can completely describe the optimization problem. Note that the all seven genes take only standardized values in the appropriate established range. These genes are listed in the table 2. It is worth mentioning here that the values of genes referring to tapered rolling bearing and radial shaft seal (genes 3, 4, 5 and 6) are according to SKF specifications.

Table 2. Genes values

No	Genes	Values
1.	Input shaft diameter, d_{ca_1}	0…63
2.	Radial shaft seal for the input shaft	0…127
3.	Tapered rolling bearing for the first shaft	0…63
4.	Tapered rolling bearing for the second shaft	0…63
5.	Tapered rolling bearing for the third shaft	0…63
6.	Radial shaft seal for the output shaft	0…127
7.	Output shaft diameter, d_{ca_2}	0…63

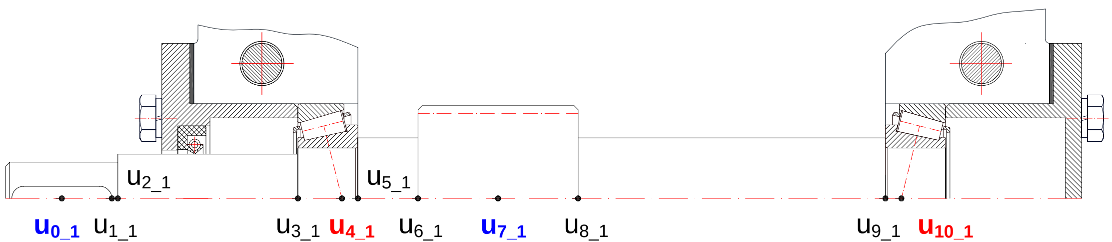

Fig. 4. The first shaft of the reducer

Objective function

The objective function chosen for this application is the mass of the three shafts sub-assemblies (including the mass of reducer shafts, of gearing and of the 6 tapered rolling bearings).

$$M_{sr} = M_{shafts} + M_{gearings} + M_{bearings} \quad (1)$$

where:

M_{shafts} – mass of the reducers shafts, [kg];
$M_{geatrings}$ – mass of the gearings, [kg];
$M_{bearings}$ – mass of the bearings, [kg].

Constraints

The solutions of the optimization program have to satisfy the listed bellow constraints. All values of these constraints have to be negative or at most zero.

C1. The shaft diameter corresponding to the radial shaft seal d_{1m_1} should be greater than the first end-shaft diameter d_{ca_1}.

$$g_1 = 1.15 \cdot d_{ca\_1} / d_{1m\_1} - 1 \quad (2)$$

C2. The bore diameter of the tapered rolling bearing d_{r_1} should be greater than the shaft diameter corresponding to radial shaft seal d_{1m_1}.

$$g_2 = d_{1m\_1} / d_{r\_1} - 1 \quad (3)$$

C3. The outer diameter of radial shaft seal d_{2m_1}, should be lower than D_{amin_1}

$$g_3 = (d_{2m\_1} + 1) / D_{a\min\_1} - 1 \quad (4)$$

C4. The pinion root diameter d_{f1} should be greater than the input shaft collar diameter d_{bmin_1}.

$$g_4 = d_{b\min\_1} / d_{f1} - 1 \quad (5)$$

C5. The maximum value of von Misses equivalent stress experienced by the first shaft should be inferior to the allowable bending stress.

$$g_5 = \sigma_{e\_1}(x)/\sigma_{aiIII} - 1 \qquad (6)$$

C6. The fatigue strength coefficient of the first shaft must be greater or equal to the allowable fatigue strength coefficient for the shaft.

$$g_6 = c_a/\min(CSO_i(P)) - 1 \qquad (7)$$

where:

$CSO_i(P)$ – function which returns the value of the fatigue strength coefficient in section i.

C7. The bending deflection of the first shaft in section u_{0_1} (fig. 4) should be less or equal to the allowable deflection bending of the shaft.

$$g_7 = \delta(u_{0\_1})/\delta_a - 1 \qquad (8)$$

C8. The bending deflection of the first shaft in section u_{7_1} (fig. 4) should be less or equal to the allowable deflection bending of the shaft.

$$g_8 = \delta(u_{7\_1})/\delta_a - 1 \qquad (9)$$

C9. The deflection at the supporting point angle of the first shaft in section u_{4_1} (fig. 4) should be less or equal to the allowable deflection at the supporting point angle of the shaft.

$$g_9 = \varphi(u_{4\_1})/\varphi_a - 1 \qquad (10)$$

C10. The deflection at the supporting point angle of the first shaft in section u_{10_1} (fig. 4) should be less or equal to the allowable deflection at the supporting point angle of the shaft.

$$g_{10} = \varphi(u_{10\_1})/\varphi_a - 1 \qquad (11)$$

C11. The torsion angle of the first shaft must be less or equal to the allowable torsion angle of the shaft.

$$g_{11} = \theta_{\_1}/\theta_a - 1 \qquad (12)$$

C12. The basic rating life must be greater to the imposed (accepted) basic rating life of the bearing.

$$g_{12} = L_{h\_nec}/L_{h\_1} - 1 \qquad (13)$$

C13. The bearing stress between key and the key way of the first end-shaft should be lower to the allowable bearing stress.

$$g_{13} = \sigma_{s\_1}/\sigma_{sa} - 1 \qquad (14)$$

C14. The shear stress of key of the input end shaft should be lower to the allowable shear stress.

$$g_{14} = \tau_{f\_1}/\tau_{fa} - 1 \qquad (17)$$

C15 Between the exterior rings of the tapered rolling bearing belonging to neighbor shafts should be at least 15 mm.

$$g_{15} = (30 + D_{r\_1} + D_{r\_2})/2/a_{w\_1} - 1 \qquad (29)$$

C16. The root diameter d_{f2} of the helical gear z_2 should be greater than its hub diameter d_{hubz2}.

$$g_{16} = d_{hubz2}/d_{f2} - 1 \qquad (19)$$

C17. The pinion root diameter d_{f3} should be greater than the second shaft collar diameter d_{ur_2}.

$$g_{17} = d_{ur\_2}/d_{f3} - 1 \qquad (8)$$

C18. The maximum value of von Misses equivalent stress suffered by the second shaft should be inferior to the allowable bending stress.

$$g_{18} = \frac{\sigma_{e\_2}(x)}{\sigma_{aiIII}} - 1 \qquad (20)$$

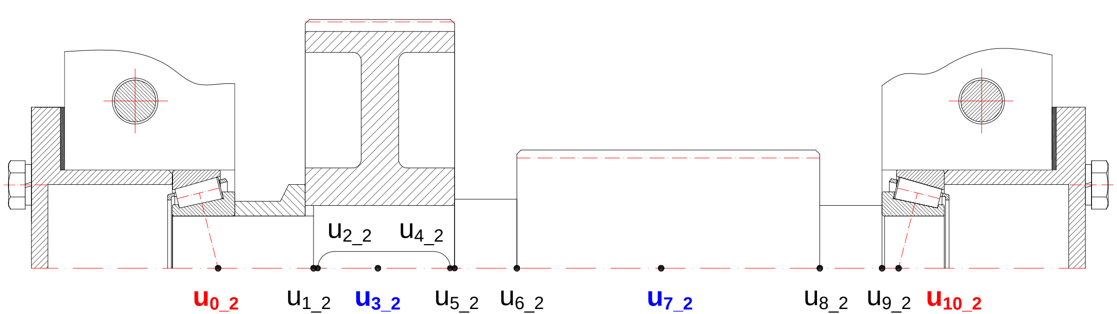

Fig. 5. The second shaft of the reducer

C19. The fatigue strength coefficient for the second shaft must be greater or equal to the allowable fatigue strength coefficient for this shaft.

$g_{19} = c_a / \min(CSO_i(P)) - 1$ (21)

C20. The bending deflection of the second shaft in section u_{3_2} (fig. 5) should be less or equal to the allowable deflection bending for the shaft.

$g_{20} = \delta(u_{3\_2}) / \delta_a - 1$ (22)

C21. The bending deflection of the second shaft in section u_{7_2} (fig. 5) should be less or equal to the allowable deflection bending for the shaft.

$g_{21} = \delta(u_{7\_2}) / \delta_a - 1$ (23)

C22. The deflection at the supporting point angle of the second shaft in section u_{0_2} (fig. 5) should be less or equal to the allowable deflection at the supporting point angle for the shaft.

$g_{22} = \varphi(u_{0\_2}) / \varphi_a - 1$ (24)

C23. The deflection at the supporting point angle of the second shaft in section u_{10_2} (fig. 5) should be less or equal to the allowable deflection at the supporting point angle for the shaft.

$g_{23} = \varphi(u_{10\_2}) / \varphi_a - 1$ (25)

C24. The torsion angle of the second shaft must be less or equal to the allowable torsion angle of the shaft.

$g_{24} = \theta_{\_2} / \theta_a - 1$ (26)

C25. The basic rating life must be greater to the imposed (accepted) basic rating life of the bearing.

$g_{25} = L_{h\_nec} / L_{h\_2} - 1$ (27)

C26. The bearing stress between key (used for the assembly of the helical gear z_2) and the key way of the second shaft should be lower to the allowable bearing stress.

$g_{26} = \sigma_{s\_2} / \sigma_{sa} - 1$ (28)

C27. The shear stress of key (used for the assembly of the helical gear) of the second shaft should be lower to the allowable shear stress.

$g_{27} = \tau_{f\_2} / \tau_{fa} - 1$ (29)

C28. Between the exterior rings of the tapered rolling bearing should be at least 15 mm.

$g_{28} = \left(30 + D_{r\_2} + D_{r\_3}\right) / 2 / a_{w\_2} - 1$ (29)

C29. The radial shaft seal diameter d_{1m_3}, should be greater than the output shaft diameter d_{ca_3}.

$g_{29} = 1.15 \cdot d_{ca\_3} / d_{1m\_3} - 1$ (30)

C30. The radial shaft seal exterior diameter d_{2m_3}, should be lower than D_{amin_3}.

$g_{30} = \left(d_{2m\_3} + 1\right) / D_{a\min\_3} - 1$ (31)

C31. The interior diameter of the tapered rolling bearing d_{r_3} should be greater than the radial shaft seal diameter d_{2m_3}.

$g_{31} = d_{1m\_3} / d_{r\_3} - 1$ (32)

C32. The root diameter d_{f4} of the helical gear z_4 should be greater than the hub diameter d_{hubz4}.

$g_{32} = d_{hubz4} / d_{f4} - 1$ (33)

C33. The maximum value of von Misses equivalent stress in the third shaft should be inferior to the allowable bending stress.

$g_{33} = \sigma_{e\_3}(x) / \sigma_{aiIII} - 1$ (34)

C34. The fatigue strength coefficient of the third shaft must be greater or equal to the allowable fatigue strength coefficient for the shaft.

$g_{34} = c_a / \min(CSO_i(P)) - 1$ (35)

C35. The bending deflection of the third shaft in section u_{5_3} (fig. 6) should be less or equal to the allowable deflection bending for the shaft.

$g_{35} = \delta(u_{5\_3}) / \delta_a - 1$ (36)

C36. The deflection at the supporting point angle of the third shaft in section u_{0_3} (fig. 6) should be less or equal to the allowable deflection at the supporting point angle for the shaft.

$g_{36} = \varphi(u_{0\_3}) / \varphi_a - 1$ (37)

C37. The deflection at the supporting point angle of the third shaft in section u_{8_3} (fig. 6) should be less or equal to the allowable deflection at the supporting point angle for the shaft.

$g_{37} = \varphi(u_{8\_3}) / \varphi_a - 1$ (38)

C38. The torsion angle of the third shaft must be less or equal to the allowable torsion angle of the shaft.

$g_{38} = \theta_{\_3} / \theta_a - 1$ (39)

C39. The basic rating life must be greater to the imposed (accepted) basic rating life of the bearing.

$$g_{39} = L_{h\_nec} / L_{h\_3} - 1 \quad (27)$$

C40. The bearing stress between key (used for the assembly of the helical gear z_4) and the key way of the third shaft should be lower to the allowable bearing stress.

$$g_{40} = \sigma_{sz4} / \sigma_{sa} - 1 \quad (28)$$

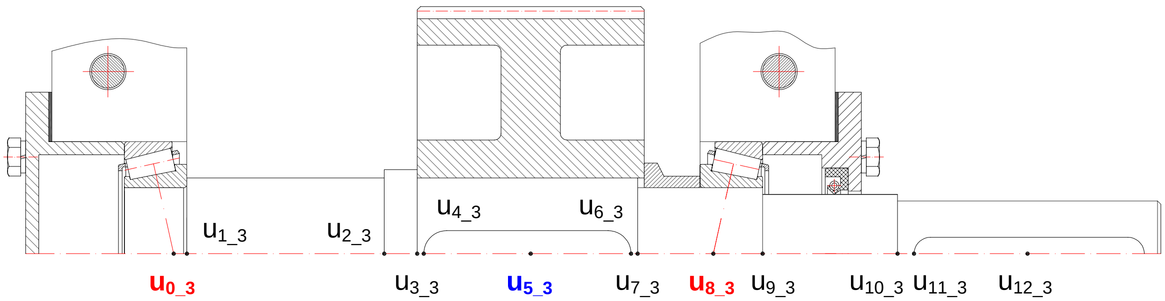

Fig. 6. The third shaft of the reducer

C41. The shear stress of key (used for the assembly of the helical gear z_4) of the third shaft should be lower to the allowable shear stress.

$$g_{41} = \tau_{fz4} / \tau_{fa} - 1 \quad (29)$$

C42. The bearing stress between key and the key way of the output end shaft should be lower to the allowable bearing stress.

$$g_{42} = \sigma_{s\_3} / \sigma_{sa} - 1 \quad (28)$$

C43. The shear stress of key of the output end-shaft should be lower to the allowable shear stress.

$$g_{43} = \tau_{f\_3} / \tau_{fa} - 1 \quad (29)$$

Results

In solving this optimization problem, our own *Kreator v*.3.2 software was used. Written in Java, Kreator is a platform that allows the assembling of all sort of evolutionary algorithms in an original manner and is in operation at the *Optimal Design Centre* of the Technical University of Cluj-Napoca, Romania

Table 3. Comparison between classical and optimal solutions

No	Genes	Values	
		Classical solution	**Optimal solution**
1	The first shaft end diameter, $d_{ca\_1} \times l_{ca\_1}$	19 × 28 mm	18 × 28 mm
2	Radial shaft seal for the input shaft	CR22×36×7 HMS5 V	CR21×35×7 HMS5 V
	d_{1m_1}	22 mm	21 mm
	d_{2m_1}	36 mm	35 mm
	b_{m_1}	7 mm	7 mm
3	Tapered rolling bearing for the first shaft	32005X/Q	32005X/Q
	d_{r_1}	25 mm	25 mm
	D_{r_1}	47 mm	47 mm
	a_{r_1}	11 mm	11 mm
	T_{r_1}	15 mm	15 mm
	C_{r_1}	11.5 mm	11.5 mm
	mass	0.11 kg	0.11 kg
4	Tapered rolling bearing for the second shaft	32005X/Q	32005X/Q
	d_{r_2}	25 mm	25 mm

	D_{r_2}	47 mm	47 mm
	a_{r_2}	11 mm	11 mm
	T_{r_2}	15 mm	15 mm
	C_{r_2}	11.5 mm	11.5 mm
	mass	0.11 kg	0.11 kg
5	Tapered rolling bearing for the third shaft	32010X/Q	32008 XTN9/Q
	d_{r_3}	50 mm	40 mm
	D_{r_3}	80 mm	68 mm
	a_{r_3}	18 mm	15 mm
	T_{r_3}	20 mm	19 mm
	C_{r_3}	15.5 mm	14.5 mm
	mass	0.37 kg	0.27 mm
6	Radial shaft seal for the third shaft	CR50×64×6 HMS5 V	CR36×52×7 HMS5 V
	d_{1m_3}	50 mm	36 mm
	d_{2m_3}	64 mm	52 mm
	b_{m_3}	6 mm	7 mm
7	The third shaft end diameter, $d_{ca\_3}\times l_{ca\_3}$	45 × 110 mm	32 × 80 mm
8	The mass of the 3 shafts sub-assemblies	14.564 kg	9.494 kg

5. CONCLUSIONS

As one can see in Table 3 the mass of the three shaft sub-assemblies (including the mass of the shafts, of gearing and of six tapered rolling bearings) representing the optimal design solution was 9.494 kg, while the classical design solution weighted 14.564 kg, which means a decrease in mass with about 34.81%.

Note that the mass of gearing is precisely computed since the area of frontal face of teeth is accurately determined.

REFERENCES

[1] Bak P., How Nature Works, Springer-Verlag, New York, 1996.

[2] Bornholdt S., and Sneppen K., Neutral Mutations and Punctuated Equilibrium, in Evolving Genetic Networks, Physical Review Letters vol. 81, no. 1, 6 July 1998, pp. 236-239.

[3] Crutchfield J.P., When Evolution is Revolution-Origins of Innovation, In Evolutionary Dynamics-Exploring the Interplay of Selection, Neutrality, Accident, and Function. J. P. Crutchfield and P. Schuster, editors. Santa Fe Institute Series in the Science of Complexity. New York, Oxford University Press, 2001.

[4] Eldredge N., and Gould S., Punctuated equilibria: An alternative to phyletic gradualism, in Schopf, T. (ed.) Models in Paleontology. San Francisco: Freeman Cooper 1972.

[5] Gould S., and Eldridge N., Punctuated equilibrium comes of age. Nature, 366: 223-227, 1993.

[6] Gould S., Wonderful Life, New York: Norton, 1989.

[7] Jonnal R., and Chemero A., Punctuation Equilibrium and Optimization: An A-life Model, in D. Blank (ed.) Proceedings of the 2000 Midwest AI and Cognitive Society Conference, Cambridge: AAAI Press, 2000.

[8] Lewis A., Abramson D., and Peachey T., An Evolutionary Programming Algorithm for Automatic Engineering Design 2003, in Fifth International Conference on Parallel Processing and Applied Mathematics, Czestochowa, Poland.

[9] Li R., Chang T., Wang J., Wei X., Multi-objective optimization design of gear reducer based on adaptive genetic algorithm, 12th International Conference on Computer Supported Cooperative Work in Design, CSCWD 2008, Xi'an, China, p. 229-233.

[10] Li X., and Symmons G.R., Optimal design of involute profile helical gears, Mechanism and machine theory, 1996, pp.717-728.

[11] Martz H.F., Michael S., Hamada C., Reese S., and Wilson A.G. Using Genetic Algorithms to Design Bayesian Reliability Experiments, in Proc. of 3 Int. Conf. on Mathematical Methods in Reliability, MMR 2002, June 17-20, 2002, Trondheim, Norway, pp. 425-428

[12] McGrath Kimberley A., Blachford S. (Editors), The Gale Encyclopedia of Science, Gale Group, 2003.

[13] Mezura E., Alternative techniques to handle constraints in evolutionary optimization, Ph.D Thesis, CIEAIPN, Mexico City, 2004.

[14] Rao S.S., Engineering optimization, 3rd Ed., New-York, Wiley, 1996.

[15] Ray T., and Saini P., Engineering design optimization using a swarm with an intelligent information sharing among individuals, IEEE Transactions on Evolutionary Computation, Vol. 7, No.4, 2003 pp.391-392.